

MARK SCHEME for the May/June 2013 series

1348 FURTHER MATHEMATICS

1348/01

Paper 1 (Further Pure Mathematics),
maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

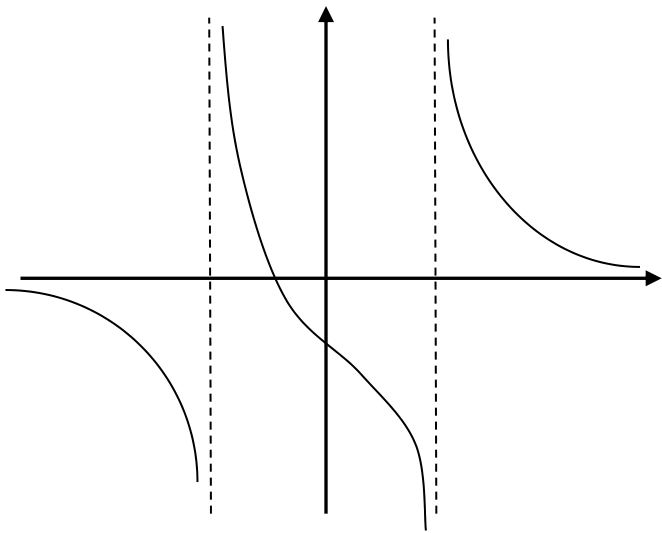
Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, Pre-U, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
	Pre-U – May/June 2013	1348	01

1		$x^2 - 6x + 12 \equiv (x - 3)^2 + 3$ $\int_2^6 \frac{1}{(\sqrt{3})^2 + (x-3)^2} dx = \left[\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x-3}{\sqrt{3}} \right) \right]_2^6 \quad \text{M1 } \tan^{-1} \quad \text{1st A1 } \left(\frac{x-3}{\sqrt{3}} \right)$ $= \frac{1}{\sqrt{3}} \left(\frac{\pi}{3} - \left(-\frac{\pi}{6} \right) \right) = \frac{\pi}{2\sqrt{3}}$	B1 M1 A1 A1 [4]
2		$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 + \dots \quad \text{from the Formula Book}$ $\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5 - \dots$ $\frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) = \frac{1}{2} \{ \ln(1+x) - \ln(1-x) \}$ $= \frac{1}{2} \times 2 \left\{ x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \dots \right\} = \tanh^{-1} x \quad \text{from the Formula Book}$ $\ln(1+x)$ valid for $-1 < x \leq 1$ and so $\ln(1-x)$ is valid for $-1 \leq x < 1$ so LHS valid for $-1 < x < 1$, which matches the range for RHS	B1 M1 A1 B1 [4]
3	(i)	$\frac{dy}{dx} = \frac{(x^2 - 4)1 - (x+1).2x}{(x^2 - 4)^2} \quad \text{Use of quotient rule; correct unsimplified}$ $= -\frac{(x^2 + 2x + 4)}{(x^2 - 4)^2} = -\frac{(x+1)^2 + 3}{(x^2 - 4)^2} \quad \text{or clear explanation this is } < 0$ ALT: $y = \frac{3/4}{x-2} + \frac{1/4}{x+2} \Rightarrow \frac{dy}{dx} = \frac{-3/4}{(x-2)^2} + \frac{-1/4}{(x+2)^2} < 0$	M1 A1 E1 [3]

Page 3	Mark Scheme	Syllabus	Paper
	Pre-U – May/June 2013	1348	01

3	(ii)	Asymptotes $y = 0$ Stated or clear from graph $x = \pm 2$ Stated or clear from graph Crossing-points $(0, -\frac{1}{4})$ and $(-1, 0)$ Noted or clearly shown on graph  3 regions All correct (incl. no TPs)	B1 B1 B1 B1 M1 A1 [6]
4	(i)	$\mathbf{d}_1 \times \mathbf{d}_2$ attempted $= 14\mathbf{i} + 35\mathbf{j} - 21\mathbf{k}$ (ALT: Use of 2 scalar prods. & attempt to get 2 components in terms of the 3rd)	M1 A1 [2]
	(ii)	Sh. Dist. $= (\mathbf{b} - \mathbf{a}) \cdot \hat{\mathbf{n}}$ $(\mathbf{b} - \mathbf{a}) = \pm(\mathbf{i} + 3\mathbf{j} - 7\mathbf{k})$ $\hat{\mathbf{n}} = \frac{1}{\sqrt{38}}(2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ ft $= \frac{1}{\sqrt{38}}(2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}) \cdot (\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}) = \frac{1}{\sqrt{38}} 2 + 15 + 21$ ft scalar prod. $= \sqrt{38}$ cao ALT: Solving $\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 10 \end{pmatrix} - \mu \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix} = k \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$ to find closest points on line, $(3, 6, 7)$ from $\lambda = 1$ and $(1, 1, 10)$ from $\mu = 0$ giving $k = 1$ and Sh.D. $= \sqrt{38}$	M1 B1 B1 B1 A1 [5]

Page 4	Mark Scheme	Syllabus	Paper
	Pre-U – May/June 2013	1348	01

5	(i)	$z^n - \frac{1}{z^n} = (\cos n\theta + i \sin n\theta) - (\cos[-n\theta] - i \sin[-n\theta])$ De Moivre's Thm. used for at least z^n $= \cos n\theta + i \sin n\theta - (\cos n\theta - i \sin n\theta) = 2i \sin n\theta$ Given answer obtained from 2 correct uses of de Moivre's Thm. and correct trig.	M1
	(ii)	$\left(z - \frac{1}{z}\right)^5 = 32i \sin^5 \theta$ $= z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$ Use of binomial expansion $= \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$ Pairing up terms $= 2i \sin 5\theta - 10i \sin 3\theta + 20i \sin \theta$ Use of (i)'s result ($\times 3$) $\Rightarrow \sin^5 \theta = \frac{1}{16} \sin 5\theta - \frac{5}{16} \sin 3\theta + \frac{5}{8} \sin \theta$	A1 [2] M1 M1 M1 M1 A1 [5]
6	(i)	$r = 1 + r \sin \theta \Rightarrow \sqrt{x^2 + y^2} = 1 + y$ Squaring and cancelling: $x^2 + y^2 = y^2 + 2y + 1 \Rightarrow y = \frac{1}{2}(x^2 - 1)$	M1 M1
	(ii)	Parabola All correct: Crossing-points at $(\pm 1, 0)$ and $(0, -\frac{1}{2})$	A1
	(iii)	$\int_{\pi}^{2\pi} \frac{1}{(1 - \sin \theta)^2} d\theta = 2 \times \int_{\pi}^{2\pi} \frac{1}{2} r^2 d\theta$ Recognition that this is related to area $= -2 \int_{-1}^1 \frac{1}{2} (x^2 - 1) dx$ Matching up with parabola-related region $= -\left[\frac{x^3}{3} - x\right]_{-1}^1 = \frac{4}{3}$ Ignore -ve answer	M1 A1 [2] M1 M1 A1 [3]

Page 5	Mark Scheme	Syllabus	Paper
	Pre-U – May/June 2013	1348	01

7	(i) $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$ or equivalent (ii) (a) $\alpha + \beta (= 3)$ and $\alpha\beta (= \frac{8}{9})$ substd. into (i)'s result ft $\Rightarrow \alpha^3 + \beta^3 = 19$ (b) $9t^2 - 27t + 8 = 0 \Rightarrow (3t - 1)(3t - 8) = 0 \Rightarrow \alpha, \beta = \frac{1}{3}, \frac{8}{3}$ Then $\alpha^3 + \beta^3 = 19 = \left(\frac{1}{3}\right)^3 + \left(\frac{8}{3}\right)^3$ Explicit statement required	M1 A1 [2] M1 A1 [2] M1 A1 A1 [3]
8	(i) (a) $x \in G \Rightarrow \exists x^{-1} \in G$ and pre-multiplying by this (or x in the $\Leftarrow$ case) gives the result (NB Both directions must be dealt with) (b) Since each xg_i is distinct, and there are n of them, the set xG is just a permutation of the elements of G OR mention that it is just a row of the group table and hence contains a permutation of the elements of G (ii) Multiply all elements together: $xg_1 xg_2 xg_3 \dots xg_n = g_1 g_2 g_3 \dots g_n$ (Since G is abelian) $\Rightarrow x^n \cdot (g_1 g_2 g_3 \dots g_n) = (g_1 g_2 g_3 \dots g_n)$ Since $g_1 g_2 g_3 \dots g_n$ is an element of G, it has an inverse; Pre/post-mult^g. by this inverse then gives $x^n = e$ (iii) (a) Elements may have an order which divides into (is a factor of) n (b) Because the change of the order of multns. in $g \cdot g_1 g \cdot g_2 g \cdot g_3 \dots g \cdot g_n = g^n \cdot (g_1 g_2 g_3 \dots g_n)$ is only valid in an abelian group	B1 B1 [2] B1 [1] E1 E1 E1 [3] B1 [1] B1 [1]

Page 6	Mark Scheme	Syllabus	Paper
	Pre-U – May/June 2013	1348	01

9		Reflection in $y = x \tan \frac{1}{8} \pi$: $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$ Allow $\cos(\frac{1}{4}\pi)$'s, etc. Shear // y-axis, mapping $(1, 0)$ to $(1, 2)$: $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ Rotation through $\frac{1}{4}\pi$ clockwise about O: $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ Shear // x-axis, mapping $(0, 1)$ to $(-2, 1)$: $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ Multiplying them together in this order (from right-to-left) = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Reflection in $y = x$ NB 1 Multiplying the matrices in the reverse order scores max. $4 \times \mathbf{B1} + \mathbf{M0}$; then B1 for correct $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and M1 for “Reflection” and A1 for “in x-axis” NB 2 Incorrect final matrices automatically lose the last 2 marks	B1 B1 B1 B1 M1 A1 M1 A1 [8]
10	(a)	$y = kx \cos x \Rightarrow \frac{dy}{dx} = -kx \sin x + k \cos x$ and $\frac{d^2y}{dx^2} = -kx \cos x - 2k \sin x$ Attempt at 1st and 2nd derivatives using the <i>Product Rule</i> Substituting both of these into the given DE $-kx \cos x - 2k \sin x + kx \cos x = 4 \sin x$ Comparing terms to evaluate k: $k = -2$ Aux. Eqn. $m^2 + 1 = 0$ solved $\Rightarrow m = \pm i$ Comp. Fn. is $y_C = A \cos x + B \sin x$ ft Accept $y_C = Ae^{ix} + Be^{-ix}$ here G. S. is $y = A \cos x + B \sin x - 2x \cos x$ ft provided y_P has no arb. const. & y_C has 2 Do not accept final answer involving complex numbers	M1 M1 M1 A1 M1 A1 B1 B1 [8]

Page 7	Mark Scheme	Syllabus	Paper
	Pre-U – May/June 2013	1348	01

10	(b)(i)	$x = 1, y = 2 \text{ \& } \frac{dy}{dx} = 1 \Rightarrow \left. \frac{d^2y}{dx^2} \right _{x=1} = -20$ Differentiating $\frac{d^2y}{dx^2} + y^2 \frac{dy}{dx} + xy = 5x - 19$ Use of <i>Product Rule</i> and implicit differentiation (at least once) $\Rightarrow \frac{d^3y}{dx^3} + \left\{ y^2 \frac{d^2y}{dx^2} + 2y \left(\frac{dy}{dx} \right)^2 \right\} + \left\{ x \frac{dy}{dx} + y \right\} = 5 \Rightarrow \left. \frac{d^2y}{dx^2} \right _{x=1} = 78$ FT “78” from “-20” and also from $\frac{dy}{dx}$ instead of $\left(\frac{dy}{dx} \right)^2$ (both = 1)	B1 M1 M1 A1 A1 A1 [6]
	(ii)	Use of $y = y(1) + (x-1).y'(1) + \frac{1}{2}(x-1)^2.y''(1) + \frac{1}{6}(x-1)^3.y'''(1) + \dots$ $= 2 + (x-1) - 10(x-1)^2 + 13(x-1)^3 + \dots \text{ ft}$ Substituting $x = 1.1$ into this series $\Rightarrow y(1.1) \approx 2.013 \text{ ft}$	M1 A1 M1 A1 [4]
11	(i)	$(p+iq)^2 = (p^2 - q^2) + i.2pq$ Comparing real and imaginary parts: $p^2 - q^2 = 63$ and $2pq = -16$ Solving simultaneously : $p = \pm 8, q = \mp 1$ i.e. $\pm(8-i)^2 = 63 - 16i$	B1 M1 M1 A1 [4]
	(ii)	(a) Use of $z^3 - (\alpha + \beta + \gamma)z^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)z - (\alpha\beta\gamma) = 0$ $A = 4 - 4i, B = 21 - 16i, C = 84$ i.e. $f(z) = z^3 - (4 - 4i)z^2 + (21 - 16i)z - 84 = 0$ (b) Differentiating to get $f'(z) = 3z^2 - 8(1-i)z + (21 - 16i)$ OR $3z^2 - 2Az + B = 0 \text{ ft}$ Solving $z = \frac{8-8i \pm \sqrt{64(1-2i-1)-12(21-16i)}}{6}$ using the quadratic formula $z = \frac{1}{3}(4-4i \pm \sqrt{16i-63}) = \frac{1}{3}(4-4i \pm i\sqrt{63-16i})$ Use of (i)’s result (on the right thing): $z = \frac{1}{3}(4-4i \pm i(8-i)) = \frac{5}{3} + \frac{4}{3}i$ or $1-4i$	M1 A1 A1 A1 [4] B1 M1 A1 M1 A1 [5]

Page 8	Mark Scheme	Syllabus	Paper
	Pre-U – May/June 2013	1348	01

12	(i)	$y'(x) = (2x + 1)e^{2x}, \quad y''(x) = (4x + 4)e^{2x},$ $y'''(x) = (8x + 12)e^{2x}, \quad y^{(4)}(x) = (16x + 32)e^{2x}$	B1 B1 B1 B1 [4]
	(ii)	Conjecture $\frac{d^n y}{dx^n} = (2^n x + n \cdot 2^{n-1})e^{2x}$ One mark each: coefft. of x , constant	B1 B1 [2]
	(iii)	Differentiating their conjectured expression (must be linear $\times e^{2x}$) $\frac{d^{n+1} y}{dx^{n+1}} = 2 \times (2^n x + n \cdot 2^{n-1})e^{2x} + 2^n \times e^{2x} \quad \text{FT max 1/2}$ $= (2^{n+1} x + (n+1) \cdot 2^{(n+1)-1})e^{2x} \quad \text{Shown of correct form}$ Usual induction round-up/explanation of proof, including clear demonstration that $(n+1)^{\text{th}}$ formula is in the right form.	M1 A1 A1 A1 E1 [5]
13	(i)	(a) $1 - \text{sech}^2 \theta \equiv \frac{(e^\theta + e^{-\theta})^2 - 4}{(e^\theta + e^{-\theta})^2} \equiv \frac{(e^\theta - e^{-\theta})^2}{(e^\theta + e^{-\theta})^2} \equiv \tanh^2 \theta$ shown legitimately (b) $\frac{d}{d\theta}(\tanh \theta) = \frac{(e^\theta + e^{-\theta})(e^\theta + e^{-\theta}) - (e^\theta - e^{-\theta})(e^\theta - e^{-\theta})}{(e^\theta + e^{-\theta})^2} \equiv \text{sech}^2 \theta$ from (a)	M1 A1 M1 A1 [4]
	(ii)	(a) $I_n = \int_0^\alpha \tanh^{2n-2} \theta \cdot \tanh^2 \theta \, d\theta = \int_0^\alpha \tanh^{2n-2} \theta (1 - \text{sech}^2 \theta) \, d\theta$ $= I_{n-1} - \left[\frac{\tanh^{2n-1} \theta}{2n-1} \right]_0^\alpha \Rightarrow I_{n-1} - I_n = \frac{(\tanh \alpha)^{2n-1}}{2n-1}$ ALT: $I_{n-1} - I_n = \int_0^\alpha \tanh^{2n-2} \theta (1 - \tanh^2 \theta) \, d\theta = \int_0^\alpha \tanh^{2n-2} \theta \cdot \text{sech}^2 \theta \, d\theta$ $= \left[\frac{\tanh^{2n-1} \theta}{2n-1} \right]_0^\alpha = \frac{(\tanh \alpha)^{2n-1}}{2n-1}$	 M1 M1 M1 A1 M1 M1 M1 A1 [4]

Page 9	Mark Scheme	Syllabus	Paper
	Pre-U – May/June 2013	1348	01

13	(ii)	(b) $I_0 = \int_0^{\alpha} 1 \, d\theta = \alpha = \frac{1}{2} \ln 3$ (c) $(I_{n-1} - I_n) + (I_{n-2} - I_{n-1}) + (I_{n-3} - I_{n-2}) + \dots + (I_2 - I_3) + (I_1 - I_2) + (I_0 - I_1)$ Use of the <i>method of differences</i> $= \sum_{r=1}^n \frac{(\tanh \alpha)^{2r-1}}{2r-1} = \sum_{r=1}^n \frac{\left(\frac{1}{2}\right)^{2r-1}}{2r-1}$ when $\alpha = \frac{1}{2} \ln 3$ $\Rightarrow I_0 - I_n = \sum_{r=1}^n \frac{\left(\frac{1}{2}\right)^{2r-1}}{2r-1}$ Cancellation of terms in the summation $\Rightarrow I_n = \frac{1}{2} \ln 3 - \sum_{r=1}^n \frac{\left(\frac{1}{2}\right)^{2r-1}}{2r-1}$ AG Ignoring “method of differences”, but opting for a direct iterative approach scores max 3/4 ... M0 M1 A1 A1 As $n \rightarrow \infty$, $I_n \rightarrow 0$ since $\tanh < 1$ $\Rightarrow \frac{1}{2} \ln 3 = \sum_{r=1}^{\infty} \frac{\left(\frac{1}{2}\right)^{2r-1}}{2r-1} = \frac{1}{1} + \frac{\left(\frac{1}{2}\right)^3}{3} + \frac{\left(\frac{1}{2}\right)^5}{5} + \frac{\left(\frac{1}{2}\right)^7}{7} + \dots$ $\Rightarrow \ln 3 = 1 + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 4^2} + \frac{1}{7 \cdot 4^3} + \dots = \sum_{r=0}^{\infty} \frac{1}{(2r+1)4^r}$ Ignoring “method of differences”, but opting for a direct iterative approach scores max 3/4 ... M0 M1 A1 A1	B1 [1] M1 A1 M1 A1 E1 M1 A1 [7]
----	------	--	---